

RHIC-AP-25

HIGHER ORDER MAGNET FIELD MULTipoles
APERTURE EFFECTS, AND TRACKING STUDIES

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Review of Tracking Theory (my view)

The instabilities are non-resonant;
not associated with the γ -values going
to some resonance line, $m\gamma_y + n\gamma_x = g$

The instabilities are not stochastic -
they happen quite fast.

No particular resonance dominates. Classical
non-linear theory does not apply. Effect
is complicated, and probably cannot be
described by simple analytical results.

Review of RHIC Results

Random b_K , $b_K \approx (k+1) b_0 / R^K$

$$R = 40 \text{ mm}, \quad b_0 \approx 1 \times 10^{-4}$$

$A_{SL} \approx 19 \text{ mm}$, random b_K only

$$\gamma_x \approx \gamma_y \approx 28.824$$

40

A₅₂ Multipole Breakdown

Single multipoles

Random b's only

$$\gamma_x = 28.827$$

$$\gamma_y = 28.822$$

A₅₂ (mm)

30

Seed 1

Seed 2

20

K=49

20

10

2

4

6

8

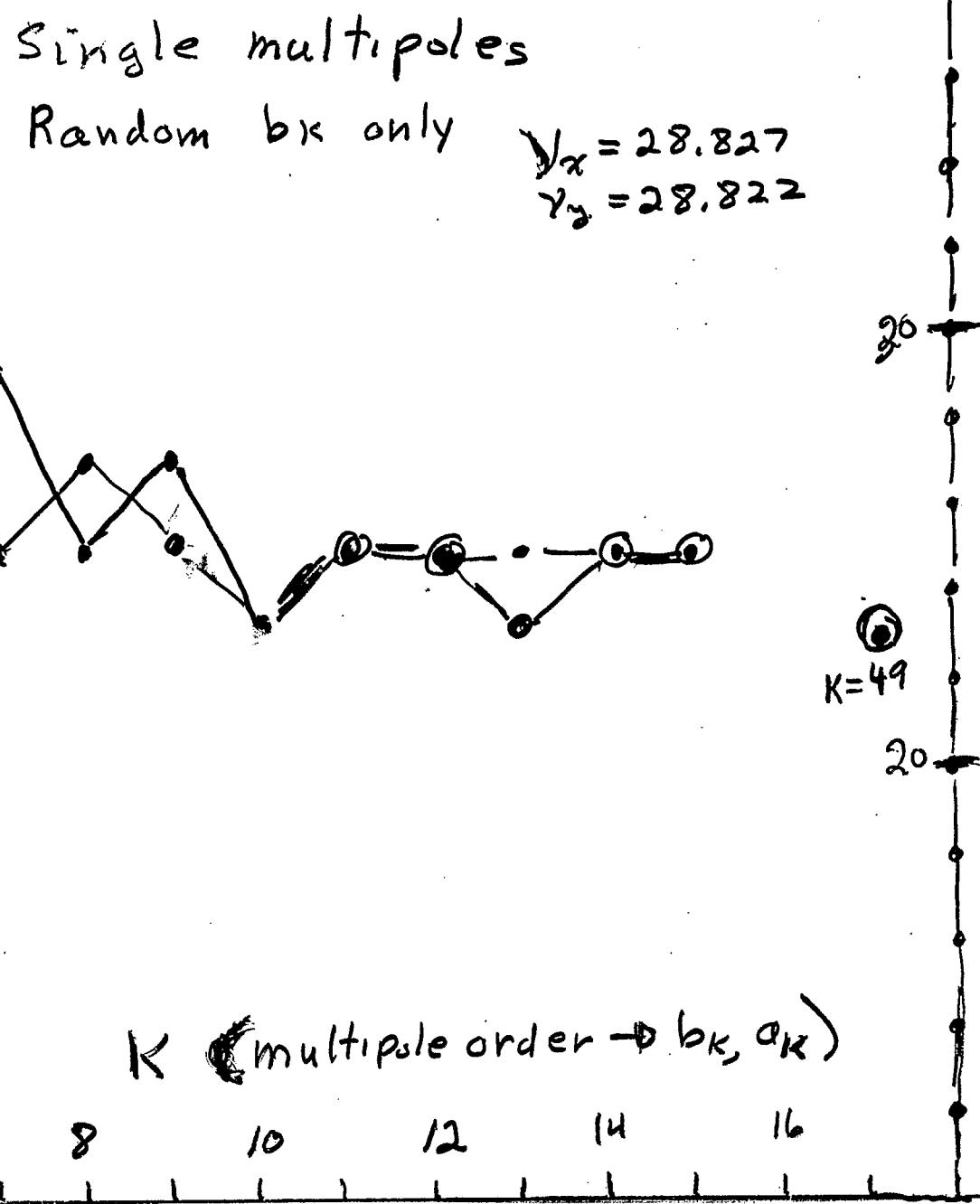
10

12

14

16

K (multipole order $\rightarrow b_K, a_K$)



(2a)

Tracking Studies seem to indicate that $A_{SL} \rightarrow$ constant $\approx 24.6\text{ mm}$ when K gets large

Is this possible?

Is it due to point multipoles being used instead of distributed multipoles?

Point Multipoles versus Distributed Multipoles

Classical N.L. Theory $\rightarrow A_{SL} \xrightarrow[K \rightarrow \infty]{} \text{constant} \approx R$
for point b_K

$$A_{SL} \xrightarrow[K \rightarrow \infty]{} 0$$

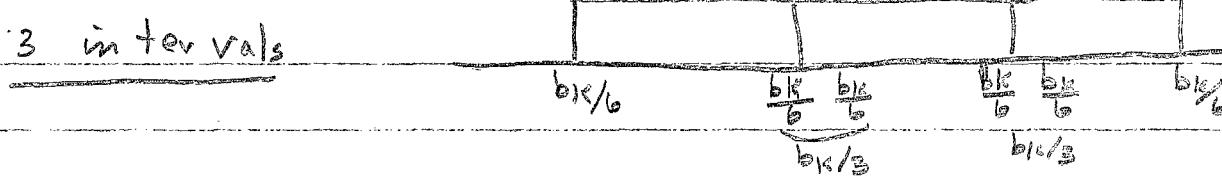
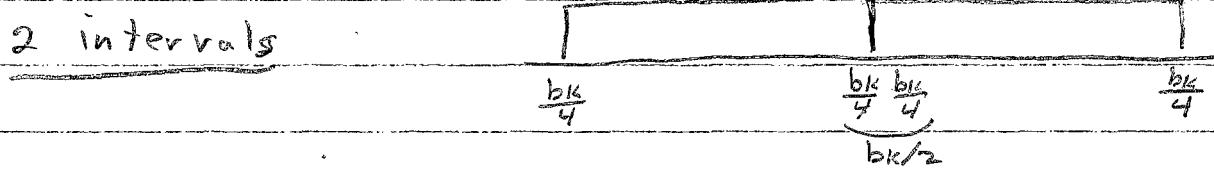
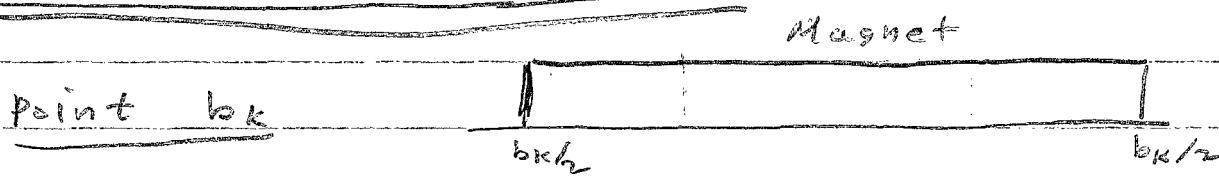
for distributed b_K

Classical theory Result

$$\gamma - \gamma_{\text{res}} \approx \underbrace{\int ds e^{i\phi_s} R \left(\frac{B}{B_0}\right)^{(K-1)/2}}_{\text{Factor}} b_K \cdot A_{SL}^{K-1}$$

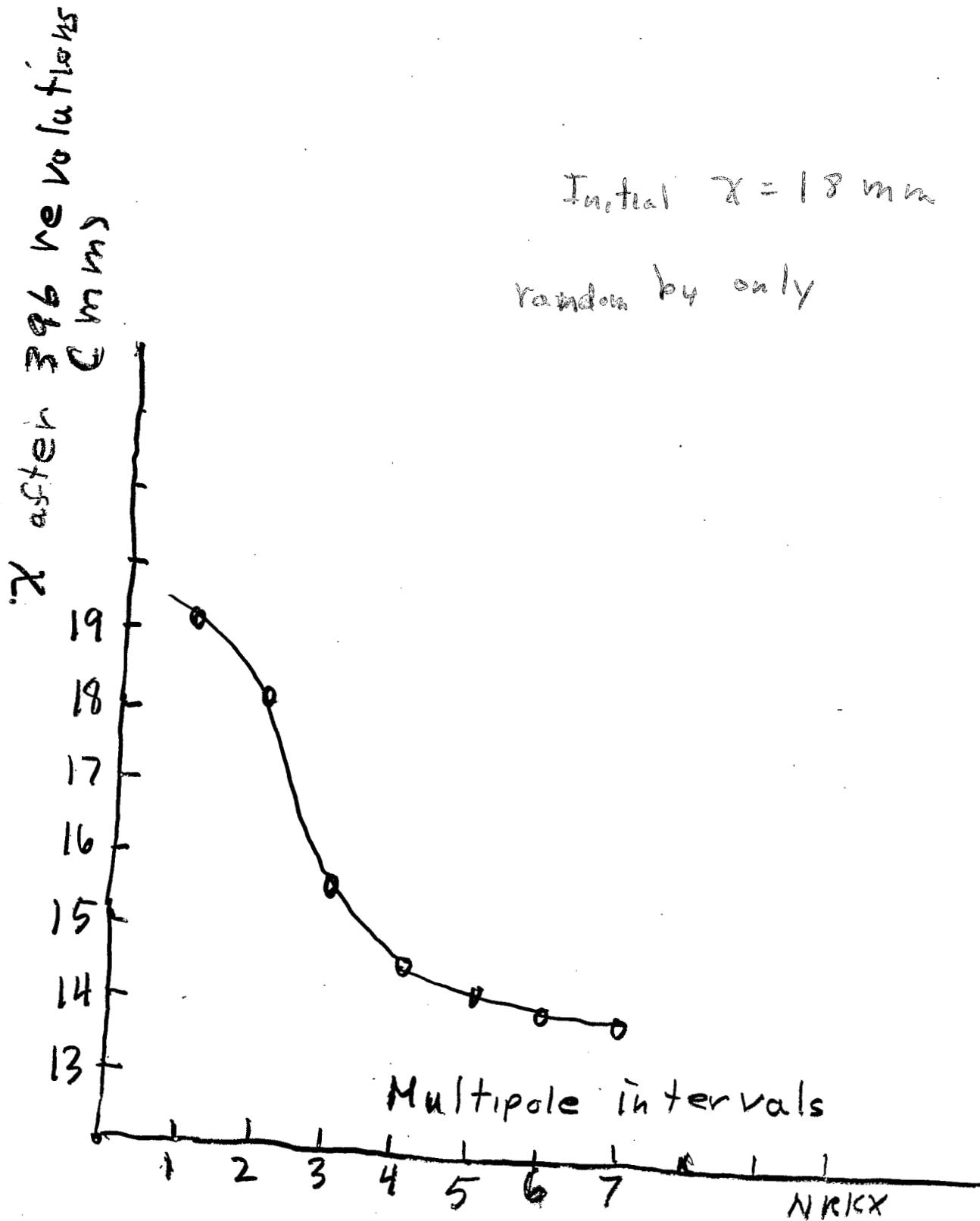
factor $\xrightarrow[K \rightarrow \infty]{} 0$ for distributed b_K

Distributed Multipoles in Tracking



Equivalent to using 2nd order Runge-Kutta.

Convergence as Multipole
Intervals are Increased



Distributed Multiple Results

Increasing the number of intervals to describe the multipoles, changes results significantly. However, A_{SL} (the stability limit) is not changed. Occasionally A_{SL} is increased by 2 mm for some runs.

Point multipoles appear to give essentially the correct result for A_{SL} .

(5)

A_{S2} for High order bk

$$A_{S2} \xrightarrow[K=0\infty]{} 24 \text{ mm}$$

for bk random case

What are the consequences?

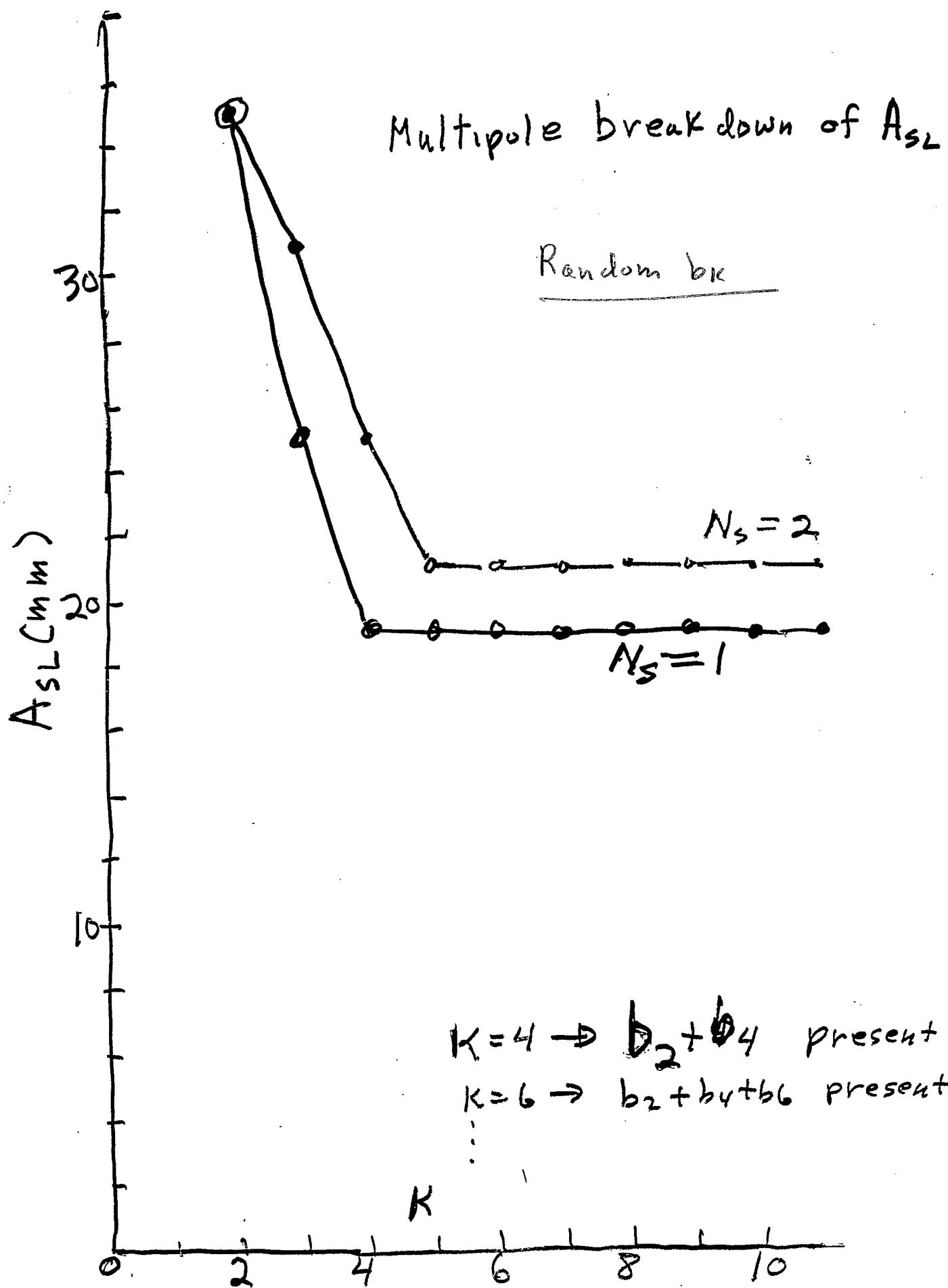
How many bk needed to determine A_{S2} ?

I don't need up to $K=50$

I just need up to $K \approx 6$

(See next slide)

6.



Higher b_k produce a wall at about 24 mm; for $x \leq 24$ mm, they have little effect. Thus if lower b_k produce $A_{SL} \approx 19$ mm, the higher b_k do not affect A_{SL} .

Higher b_k Limit A_{SL} to $A_{SL} \leq 24$ mm

Even if I correct many of the lower b_k , say for $K \lesssim 10$, I can't do better than $A_{SL} \approx 24$ mm.

Systematic bk

Possible problem: I expect
the higher systematic bk to be larger
than the higher random bk

$$bk \approx \frac{b_0}{k^4} \times k^{-4}$$

$$b_0 \approx (k+1) \times 1.1 \times 10^{-4} \text{ random bk}$$

$$b_0 \approx 300 \times 10^{-4} \text{ not unlikely
for systematic bk}$$

$b_0 \sim 1$ is possible for systematic bk

(9)

Systematic b_K , RHIC Results

$b_K, \text{ systematic}$ used (from RHIC Proposal I)

Dipole

$$R = 40\text{ mm}$$

$$K \quad b'_K \quad b_K + R^{K/10}^{-4}$$

Quads

$$ik' \quad b'_K \quad b_K + R^{K/10}^{-4}$$

K	b'_K	$b_K + R^{K/10}^{-4}$
2	17	44
4	-5.9	-38
6	1.6	17
8	-.4	-10
10	.1	7
12	-.1	-10
14	-.1	-72
16	-.13	-239
18	.080	378
20	.0030	36

$$\underline{P} = 1 \times (k+1) \text{ for random } b_K$$

Results for $\left\{ \begin{array}{l} \text{Dipoles} \\ K = 14 \rightarrow 20 \\ \text{Quads} \end{array} \right\}$ from H. Hahn Tech Note

Note, $A_{SL} \sim b_K^{\frac{1}{K}}$ or large changes in b_K

produce small changes in A_{SL} for large K

FINAL b_K , systematic

measured results

$$K \quad b_K (m^{-1}/10^{-4}) \quad b_K \cdot R^K / b^{-4}$$

$$R = 38 \text{ mm}$$

2	.99	2.2
3	-1.27	= .91
4	-1.76	= 3.8
5	-1.05	= .38 -
6	6.69	76.
7	.02	.34
8	-15.69	-404
9	.01	.38
10	5.25	302
12	-1.1	-142
14	.12	35.
16		

Aperture Results (Tracking Results)

A_{SL} including b_K , systematic

For $b_{K,sys}(k=1 \rightarrow 20) + b_{K,ran}(k=1 \rightarrow 10)$

$$A_{SL} = 15 \text{ mm}$$

For $b_{K,sys}(k=1 \rightarrow 20)$; no $b_{K,ran}$

$$A_{SL} = 19 \text{ mm}$$

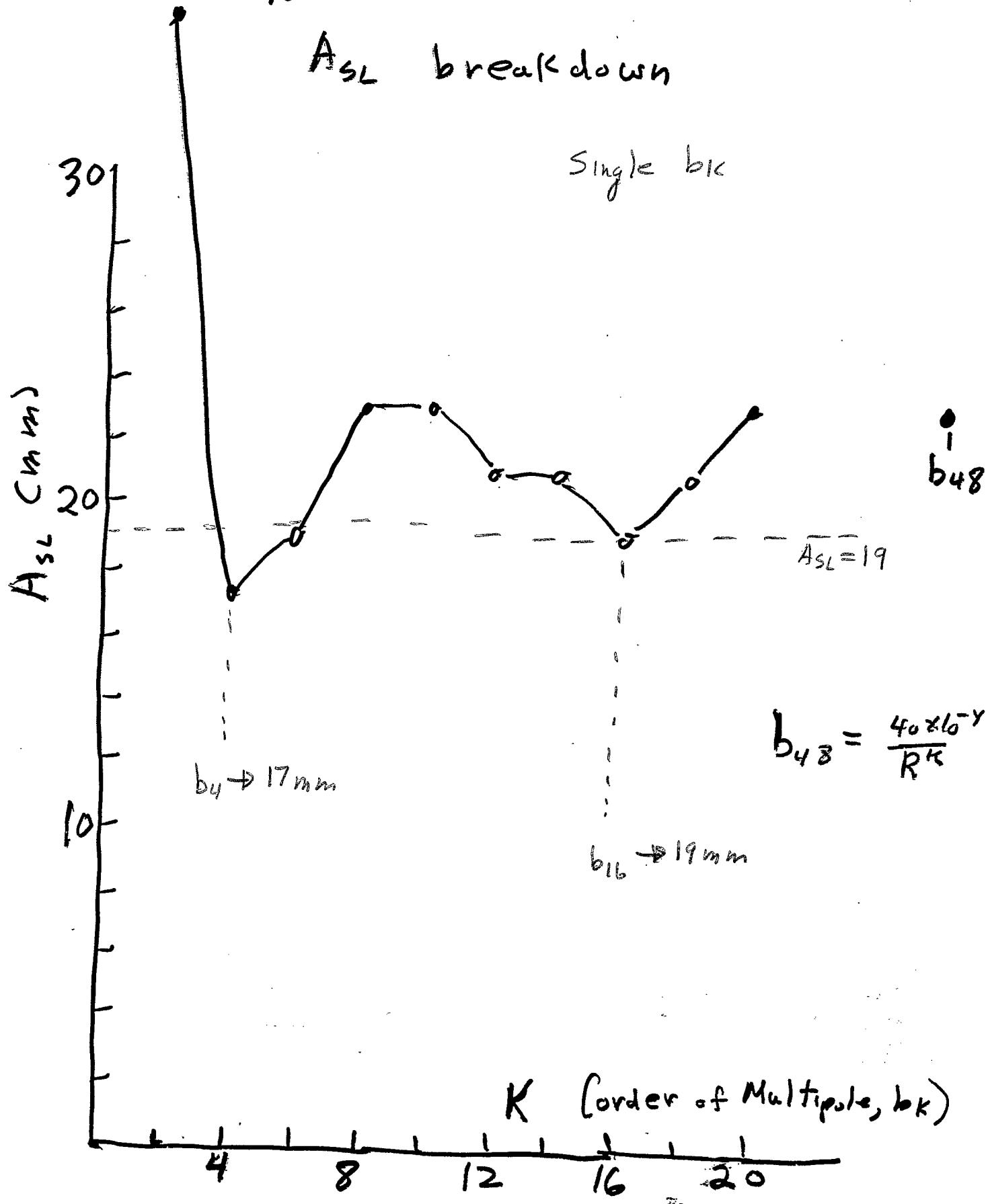
Same as $b_{K,random}$ case

b_K , systematic, multipole breakdown
(see next page)

Systematic b_K

A_{SL} breakdown

Single b_K



For $b_{k,sys}$ ($k=10 \rightarrow 20$) + $b_{k,ran}$ ($k=1 \rightarrow 10$)

$$A_{SL} = 17 \text{ mm}$$

Even if ~~$b_{k,sys}$~~ for $k=1 \rightarrow 9$
are eliminated, one gets

$$A_{SL} = 17 \text{ mm}$$

There is the possibility, and some indication, that proper choice of lower b_k ($b_{k,lo}$) can increase A_{SL}

Non-point $b_{k,sys}$ tests

Distributed $b_{k,sys}$ produce only small changes in the results.

FNAL Aperture Results

$b_{k,sys}$ all alone $\rightarrow A_{SL} = 21 \text{ mm } (\beta_x = 100)$

addition of $b_{k,ran}$ reduces A_{SL} to

$A_{SL} = 19 \text{ mm } (\beta_x = 100; \gamma \approx 0 \text{ results})$

Tracking results of Gelfand and Willeke

Willeke says Tevatron aperture is largely due to $b_{k,sys}$. I think both, $b_{k,sys}$ and $b_{k,ran}$, are important

Note, Tevatron A_{SL} for $\pi \approx y$
may be $A_{SL} \approx 15 \text{ mm}$.

Conclusions for RHIC Magnets

1) $A_{SL} \approx 17$ mm may result from systematic b_K .

2) Possibility of choosing lower b_K to improve A_{SL} .
 $b_K = 0$ is not necessarily optimum solution.

3) Watch out for very large higher b_K (b_{16} etc., etc.)